

Spring 2019 Math 245 Exam 3

Please read the following directions:

Please write legibly, with plenty of white space. Please fill out the box above as legibly as possible. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will begin at 10:00 and will end at 10:50; pace yourself accordingly. Please remain quiet to ensure a good test environment for others. Good luck!

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

a. $=$ (for sets)

b. union

c. disjoint

Problem 2. Carefully define the following terms:

a. De Morgan's Law (for sets)

b. Cantor's Theorem

c. transitive

Problem 3. Let R, S be sets with $R \setminus S = S \setminus R$. Prove that $R \subseteq S$.

Problem 4. Prove or disprove: For all sets R, S, T satisfying $R \subseteq S$, $S \subseteq T$, and $T \subseteq R$, we must have $R = S$.

Problem 5. Prove or disprove: For all sets R, S , we have $R \times S = S \times R$.

Problem 6. Prove or disprove: For all sets R, S , we have $|R \times S| = |S \times R|$.

Problem 7. Consider relation $R = \{(a, b) : a^2 \geq b\}$ on \mathbb{Q} . Prove or disprove that R is reflexive.

Problem 8. Prove or disprove: For all sets R, S , we have $2^R \cup 2^S = 2^{R \cup S}$.

Problem 9. Let R, S, T be sets. Prove that $R \cap (S \cup T) \subseteq (R \cap S) \cup (R \cap T)$.
Your answer should not use any theorems about sets.

Problem 10. Consider relation $R = \{(a, b) : b \leq a \leq 3b\}$ on \mathbb{N}_0 . Compute and simplify $R \circ R$.
Your answer should not use quantifiers.